

GOALS, CONSTRAINTS, AND TRANSPARENTLY FAIR ASSIGNMENTS: A FIELD STUDY OF RANDOMIZATION DESIGN IN THE UEFA CHAMPIONS LEAGUE

ONLINE SUPPLEMENTARY MATERIAL

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CONTENTS

1. Expected assignment matrices for the ULC seasons 2004–2019	2
2. Fairness objective for an eight-by-eight assignment: Current versus optimal	6
2.1. Small-Dimension analysis	6
2.2. Monte-Carlo simulation exercise: Independent draws	8
2.3. Monte-Carlo simulation exercise: Correlated draws	16
3. Fairness objective for other assignments: Current versus optimal	24
3.1. Six-by-six assignment	24
3.2. Seven-by-seven assignment	25

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1. EXPECTED ASSIGNMENT MATRICES FOR THE ULC SEASONS 2004–2019

TABLE 1. Season 2004

	FC Bayern Munchen	FC Lokomotiv Moskva	RC Deportivo La Coruna	Real Sociedad de Futbol	VfB Stuttgart	FC Porto	AC Sparta Praha	RC Celta de Vigo
Olympique Lyonnais	0.	0.123	0.164	0.164	0.123	0.139	0.123	0.164
Arsenal FC	0.123	0.	0.164	0.164	0.123	0.139	0.122	0.164
AS Monaco FC	0.127	0.127	0.	0.173	0.127	0.147	0.127	0.172
Juventus	0.127	0.127	0.172	0.	0.128	0.147	0.127	0.173
Manchester United FC	0.123	0.123	0.164	0.164	0.	0.14	0.124	0.164
Real Madrid CF	0.25	0.25	0.	0.	0.25	0.	0.25	0.
Chelsea FC	0.123	0.124	0.164	0.163	0.123	0.14	0.	0.164
AC Milan	0.127	0.127	0.173	0.172	0.127	0.147	0.127	0.

TABLE 2. Season 2005

	Liverpool FC	Real Madrid CF	FC Bayern Munchen	Manchester United FC	PSV Eindhoven	FC Barcelona	SV Werder Bremen	FC Porto
AS Monaco FC	0.	0.126	0.145	0.198	0.132	0.122	0.146	0.131
Bayer 04 Leverkusen	0.251	0.	0.	0.251	0.169	0.16	0.	0.17
Juventus	0.185	0.124	0.	0.184	0.123	0.117	0.142	0.124
Olympique Lyonnais	0.198	0.128	0.145	0.	0.131	0.122	0.145	0.131
Arsenal FC	0.	0.19	0.214	0.	0.	0.182	0.215	0.2
AC Milan	0.181	0.12	0.138	0.181	0.121	0.	0.138	0.121
FC Internazionale Milano	0.185	0.123	0.142	0.185	0.124	0.117	0.	0.124
Chelsea FC	0.	0.189	0.215	0.	0.2	0.181	0.215	0.

TABLE 3. Season 2006

	FC Bayern Munchen	AFC Ajax	SV Werder Bremen	SL Benfica	PSV Eindhoven	Real Madrid CF	Chelsea FC	Rangers FC
Juventus	0.	0.131	0.133	0.132	0.129	0.192	0.154	0.128
Arsenal FC	0.153	0.	0.157	0.158	0.153	0.226	0.	0.153
FC Barcelona	0.16	0.164	0.	0.168	0.161	0.	0.188	0.16
Villarreal CF	0.16	0.164	0.167	0.	0.16	0.	0.19	0.16
AC Milan	0.129	0.132	0.133	0.132	0.	0.192	0.152	0.13
Olympique Lyonnais	0.137	0.14	0.143	0.143	0.137	0.	0.162	0.137
Liverpool FC	0.132	0.137	0.135	0.135	0.132	0.197	0.	0.132
FC Internazionale Milano	0.129	0.132	0.133	0.132	0.129	0.193	0.153	0.

TABLE 4. Season 2007

	FC Barcelona	FC Internazionale Milano	PSV Eindhoven	AS Roma	Real Madrid CF	Celtic FC	FC Porto	LOSC Lille
Chelsea FC	0.	0.152	0.124	0.164	0.158	0.124	0.123	0.156
FC Bayern Munchen	0.152	0.	0.124	0.164	0.155	0.124	0.124	0.157
Liverpool FC	0.148	0.148	0.	0.159	0.151	0.121	0.121	0.152
Valencia CF	0.	0.223	0.182	0.	0.	0.182	0.182	0.231
Olympique Lyonnais	0.181	0.181	0.147	0.197	0.	0.146	0.147	0.
Manchester United FC	0.148	0.148	0.121	0.158	0.151	0.	0.121	0.152
Arsenal FC	0.148	0.148	0.12	0.158	0.152	0.121	0.	0.152
AC Milan	0.222	0.	0.182	0.	0.232	0.183	0.182	0.

TABLE 5. Season 2008

	Liverpool FC	FC Schalke 04	Olympiacos FC	Celtic FC	Olympique Lyonnais	AS Roma	Fenerbahce SK	Arsenal FC
FC Porto	0.	0.129	0.121	0.123	0.12	0.187	0.123	0.198
Chelsea FC	0.	0.	0.179	0.182	0.179	0.278	0.182	0.
Real Madrid CF	0.18	0.119	0.	0.116	0.114	0.174	0.116	0.181
AC Milan	0.221	0.144	0.137	0.	0.137	0.	0.141	0.22
FC Barcelona	0.181	0.12	0.114	0.116	0.	0.174	0.116	0.18
Manchester United FC	0.	0.214	0.194	0.2	0.194	0.	0.199	0.
FC Internazionale Milano	0.22	0.145	0.137	0.141	0.137	0.	0.	0.22
Sevilla FC	0.198	0.13	0.12	0.123	0.12	0.187	0.123	0.

TABLE 6. Season 2009

	Chelsea FC	FC Internazionale Milano	Sporting Clube de Portugal	Club Atletico de Madrid	Villarreal CF	Olympique Lyonnais	Arsenal FC	Real Madrid CF
AS Roma	0.	0.	0.163	0.171	0.13	0.13	0.202	0.164
Panathinaikos FC	0.167	0.	0.137	0.141	0.142	0.109	0.166	0.137
FC Barcelona	0.282	0.248	0.	0.	0.	0.189	0.281	0.
Liverpool FC	0.	0.217	0.204	0.	0.216	0.16	0.	0.204
Manchester United FC	0.	0.216	0.205	0.216	0.	0.16	0.	0.203
FC Bayern Munchen	0.16	0.145	0.132	0.136	0.136	0.	0.16	0.131
FC Porto	0.198	0.175	0.	0.17	0.169	0.126	0.	0.161
Juventus	0.192	0.	0.16	0.165	0.166	0.126	0.191	0.

TABLE 7. Season 2010

	FC Bayern Munchen	PFC CSKA Moskva	AC Milan	FC Porto	Olympique Lyonnais	FC Internazionale Milano	VfB Stuttgart	Olympiacos FC
FC Girondins de Bordeaux	0.	0.152	0.194	0.154	0.	0.194	0.153	0.153
Manchester United FC	0.131	0.	0.16	0.128	0.165	0.16	0.128	0.128
Real Madrid CF	0.135	0.132	0.	0.132	0.171	0.166	0.132	0.132
Chelsea FC	0.131	0.128	0.16	0.	0.164	0.16	0.129	0.128
ACF Fiorentina	0.206	0.199	0.	0.198	0.	0.	0.198	0.199
FC Barcelona	0.135	0.132	0.167	0.131	0.171	0.	0.132	0.132
Sevilla FC	0.131	0.128	0.16	0.128	0.165	0.16	0.	0.128
Arsenal FC	0.131	0.128	0.159	0.129	0.164	0.16	0.128	0.

TABLE 8. Season 2011

	FC Internazionale Milano	Olympique Lyonnais	Valencia CF	FC Kobenhavn	AS Roma	Olympique de Marseille	AC Milan	Arsenal FC
Tottenham Hotspur FC	0.	0.153	0.227	0.156	0.151	0.158	0.155	0.
FC Schalke 04	0.121	0.	0.175	0.12	0.118	0.12	0.12	0.226
Manchester United FC	0.169	0.161	0.	0.168	0.163	0.17	0.168	0.
FC Barcelona	0.147	0.143	0.	0.	0.143	0.147	0.147	0.274
FC Bayern Munchen	0.121	0.118	0.175	0.119	0.	0.121	0.12	0.225
Chelsea FC	0.158	0.152	0.227	0.155	0.152	0.	0.156	0.
Real Madrid CF	0.147	0.142	0.	0.147	0.143	0.147	0.	0.274
FC Shakhtar Donetsk	0.137	0.13	0.196	0.134	0.131	0.137	0.134	0.

TABLE 9. Season 2012

	SSC Napoli	PFC CSKA Moskva	FC Basel 1893	Olympique Lyonnais	Bayer 04 Leverkusen	Olympique de Marseille	FC Zenit	AC Milan
FC Bayern Munchen	0.	0.171	0.158	0.158	0.	0.158	0.157	0.198
FC Internazionale Milano	0.	0.	0.193	0.193	0.229	0.192	0.193	0.
SL Benfica	0.164	0.136	0.	0.129	0.154	0.129	0.129	0.159
Real Madrid CF	0.165	0.137	0.128	0.	0.154	0.128	0.129	0.16
Chelsea FC	0.17	0.139	0.131	0.132	0.	0.132	0.132	0.164
Arsenal FC	0.165	0.137	0.129	0.128	0.153	0.	0.129	0.16
APOEL FC	0.164	0.136	0.129	0.129	0.153	0.129	0.	0.16
FC Barcelona	0.172	0.144	0.132	0.132	0.157	0.132	0.132	0.

TABLE 10. Season 2013

	FC Porto	Arsenal FC	AC Milan	Real Madrid CF	FC Shakhtar Donetsk	Valencia CF	Celtic FC	Galatasaray AS
Paris Saint-Germain	0.	0.134	0.144	0.183	0.117	0.184	0.121	0.116
FC Schalke 04	0.117	0.	0.146	0.187	0.119	0.187	0.124	0.12
Malaga CF	0.189	0.217	0.	0.	0.194	0.	0.209	0.192
Borussia Dortmund	0.122	0.142	0.155	0.	0.125	0.201	0.132	0.125
Juventus	0.135	0.156	0.	0.215	0.	0.216	0.141	0.137
FC Bayern Munchen	0.122	0.141	0.154	0.202	0.125	0.	0.132	0.124
FC Barcelona	0.183	0.21	0.235	0.	0.186	0.	0.	0.187
Manchester United FC	0.133	0.	0.166	0.212	0.135	0.213	0.141	0.

TABLE 11. Season 2014

	Bayer 04 Leverkusen	Galatasaray AS	Olympiacos FC	Manchester City FC	FC Schalke 04	Arsenal FC	FC Zenit	AC Milan
Manchester United FC	0.	0.172	0.173	0.	0.31	0.	0.172	0.172
Real Madrid CF	0.173	0.	0.103	0.172	0.173	0.173	0.104	0.103
Paris Saint-Germain	0.172	0.103	0.	0.173	0.173	0.173	0.103	0.103
FC Bayern Munchen	0.	0.173	0.173	0.	0.	0.309	0.172	0.173
Chelsea FC	0.309	0.173	0.173	0.	0.	0.	0.173	0.172
Borussia Dortmund	0.	0.172	0.172	0.311	0.	0.	0.173	0.173
Club Atletico de Madrid	0.173	0.104	0.103	0.172	0.172	0.172	0.	0.104
FC Barcelona	0.173	0.103	0.103	0.172	0.173	0.173	0.103	0.

TABLE 12. Season 2015

	Juventus	FC Basel 1893	Bayer 04 Leverkusen	Arsenal FC	Manchester City FC	Paris Saint-Germain	FC Schalke 04	FC Shakhtar Donetsk
Club Atletico de Madrid	0.	0.11	0.177	0.143	0.143	0.127	0.19	0.11
Real Madrid CF	0.11	0.	0.177	0.142	0.143	0.128	0.189	0.111
AS Monaco FC	0.135	0.135	0.	0.177	0.177	0.	0.24	0.135
Borussia Dortmund	0.18	0.181	0.	0.	0.25	0.209	0.	0.181
FC Bayern Munchen	0.18	0.181	0.	0.25	0.	0.209	0.	0.181
FC Barcelona	0.113	0.112	0.18	0.145	0.145	0.	0.193	0.112
Chelsea FC	0.172	0.171	0.288	0.	0.	0.198	0.	0.171
FC Porto	0.11	0.11	0.178	0.143	0.142	0.129	0.189	0.

TABLE 13. Season 2016

	Paris Saint-Germain	PSV Eindhoven	SL Benfica	Juventus	AS Roma	Arsenal FC	FC Dynamo Kyiv	KAA Gent
Real Madrid CF	0.	0.129	0.129	0.131	0.129	0.193	0.158	0.131
VfL Wolfsburg	0.129	0.	0.128	0.132	0.129	0.192	0.158	0.132
Club Atletico de Madrid	0.129	0.128	0.	0.132	0.129	0.193	0.159	0.131
Manchester City FC	0.159	0.16	0.16	0.	0.16	0.	0.198	0.164
FC Barcelona	0.129	0.129	0.129	0.132	0.	0.193	0.158	0.131
FC Bayern Munchen	0.137	0.137	0.137	0.143	0.136	0.	0.169	0.14
Chelsea FC	0.164	0.164	0.164	0.172	0.165	0.	0.	0.171
FC Zenit	0.153	0.153	0.154	0.158	0.153	0.23	0.	0.

TABLE 14. Season 2017

	Paris Saint-Germain	SL Benfica	Manchester City FC	FC Bayern Munchen	Bayer 04 Leverkusen	Real Madrid CF	FC Porto	Sevilla FC
Arsenal FC	0.	0.129	0.	0.165	0.158	0.216	0.133	0.199
SSC Napoli	0.127	0.	0.158	0.136	0.13	0.176	0.109	0.164
FC Barcelona	0.204	0.174	0.	0.231	0.212	0.	0.179	0.
Club Atletico de Madrid	0.197	0.169	0.252	0.	0.209	0.	0.173	0.
AS Monaco FC	0.	0.126	0.185	0.162	0.	0.208	0.127	0.192
Borussia Dortmund	0.188	0.161	0.236	0.	0.	0.	0.164	0.251
Leicester City FC	0.151	0.128	0.	0.162	0.154	0.21	0.	0.195
Juventus	0.133	0.113	0.168	0.145	0.137	0.19	0.114	0.

TABLE 15. Season 2019

	Club Atletico de Madrid	Tottenham Hotspur FC	Liverpool FC	FC Schalke 04	AFC Ajax	Olympique Lyonnais	AS Roma	Manchester United FC
Borussia Dortmund	0.	0.175	0.175	0.	0.14	0.175	0.161	0.174
FC Barcelona	0.	0.	0.173	0.186	0.136	0.174	0.159	0.173
Paris Saint-Germain	0.186	0.174	0.	0.18	0.132	0.	0.155	0.173
FC Porto	0.163	0.146	0.147	0.	0.117	0.146	0.135	0.145
FC Bayern Munchen	0.184	0.166	0.167	0.	0.	0.166	0.152	0.165
Manchester City FC	0.282	0.	0.	0.273	0.208	0.	0.237	0.
Real Madrid CF	0.	0.171	0.17	0.183	0.135	0.171	0.	0.171
Juventus	0.185	0.168	0.169	0.179	0.131	0.168	0.	0.

2. FAIRNESS OBJECTIVE FOR AN EIGHT-BY-EIGHT ASSIGNMENT: CURRENT VERSUS OPTIMAL

2.1. Small-Dimension analysis.

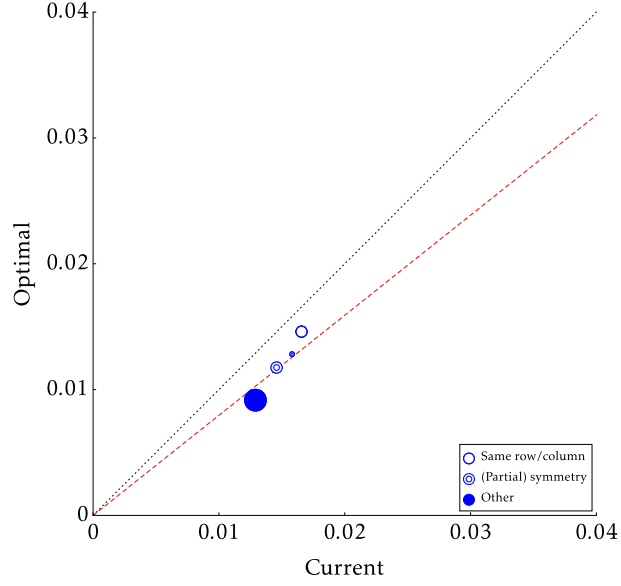


FIGURE 1. Number of constraints: $C = 2$. Red dashed line indicates fitted linear relationship

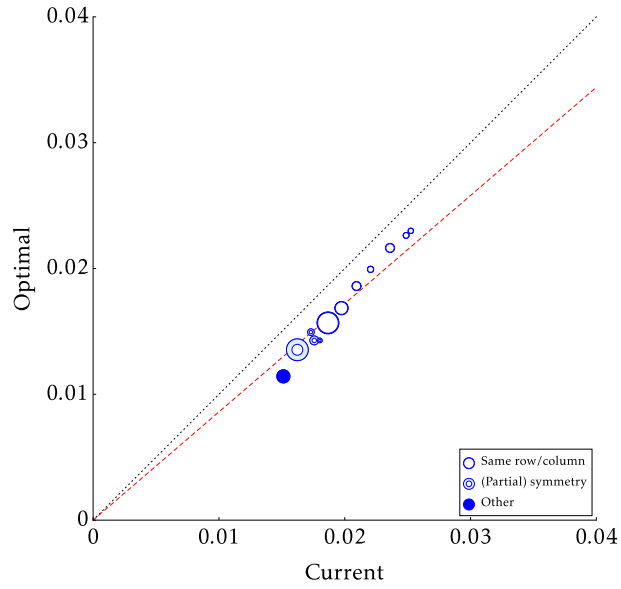


FIGURE 2. Number of constraints: $C = 3$. Red dashed line indicates fitted linear relationship

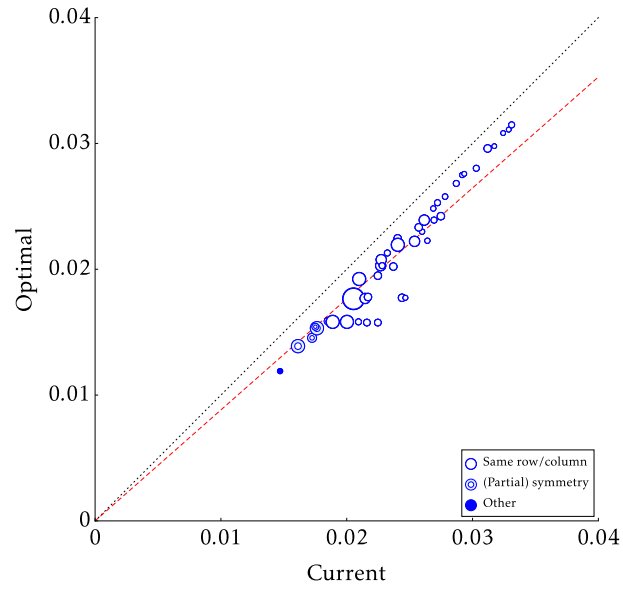


FIGURE 3. Number of constraints: $C = 4$. Red dashed line indicates fitted linear relationship

2.2. Monte-Carlo simulation exercise: Independent draws.

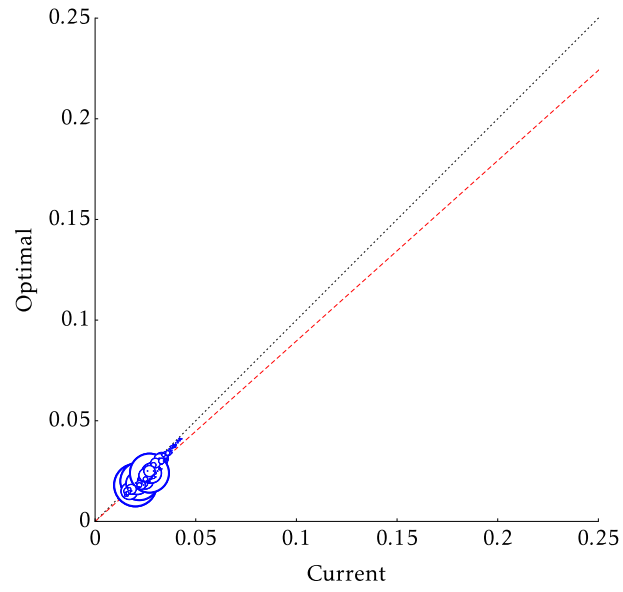


FIGURE 4. Number of constraints: $C = 5$. Red dashed line indicates fitted linear relationship

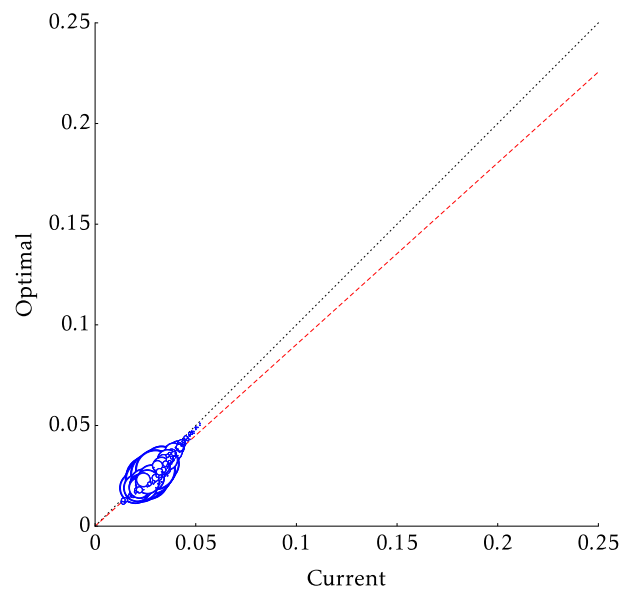


FIGURE 5. Number of constraints: $C = 6$. Red dashed line indicates fitted linear relationship

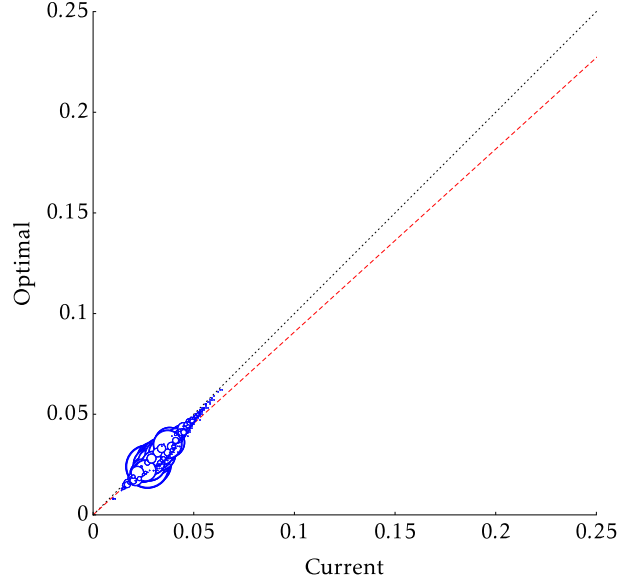


FIGURE 6. Fairness objective under independent constraint structures: Optimal versus current, $C = 7$. Red dashed line indicates fitted linear relationship

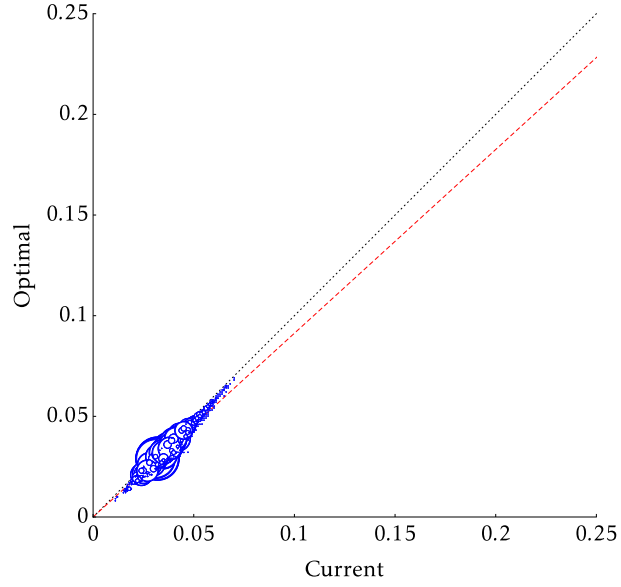


FIGURE 7. Number of constraints: $C = 8$. Red dashed line indicates fitted linear relationship

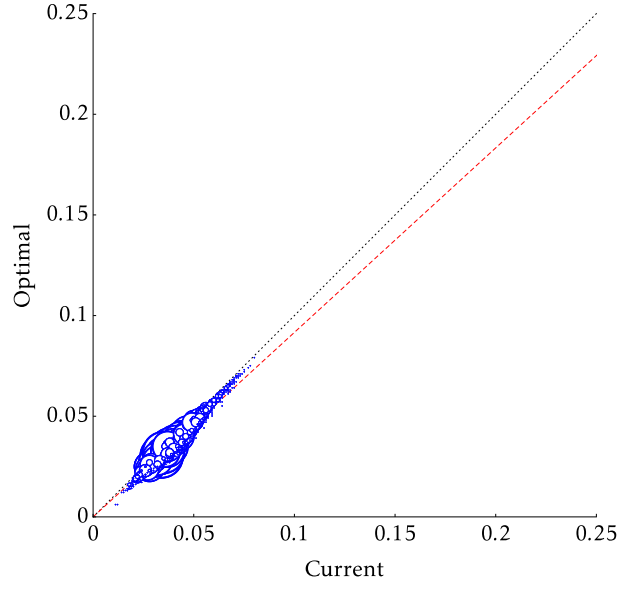


FIGURE 8. Number of constraints: $C = 9$. Red dashed line indicates fitted linear relationship

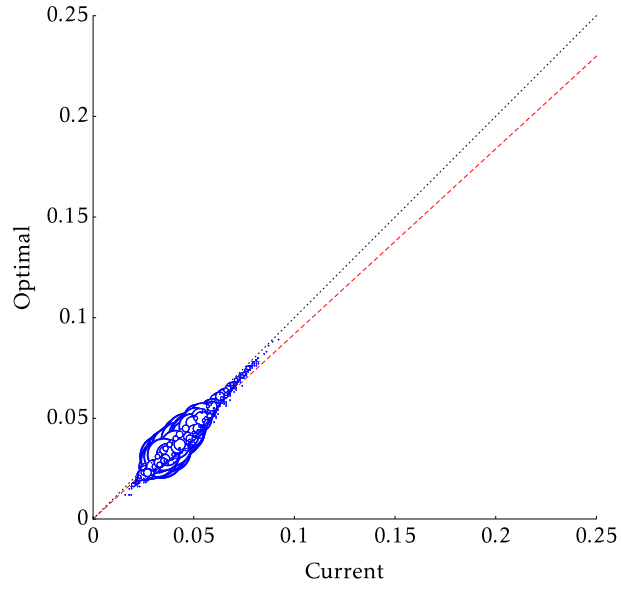


FIGURE 9. Number of constraints: $C = 10$. Red dashed line indicates fitted linear relationship

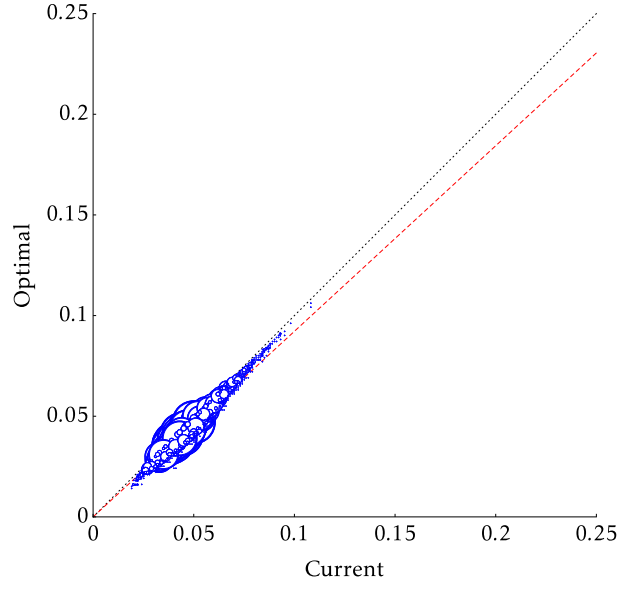


FIGURE 10. Number of constraints: $C = 11$. Red dashed line indicates fitted linear relationship

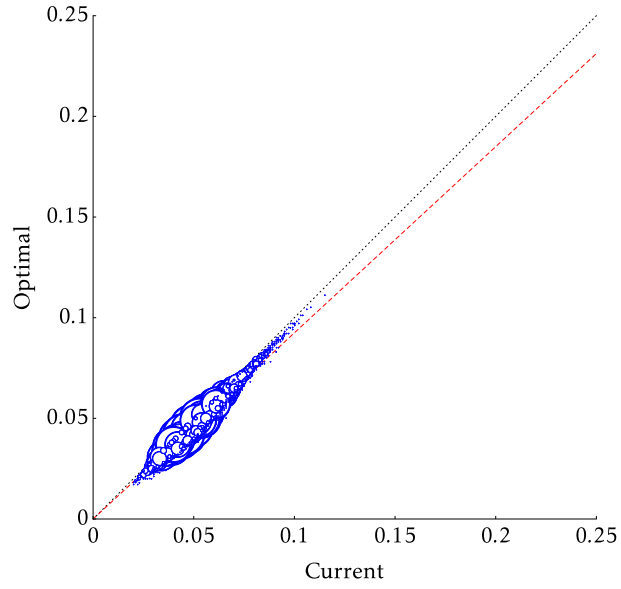


FIGURE 11. Number of constraints: $C = 12$. Red dashed line indicates fitted linear relationship

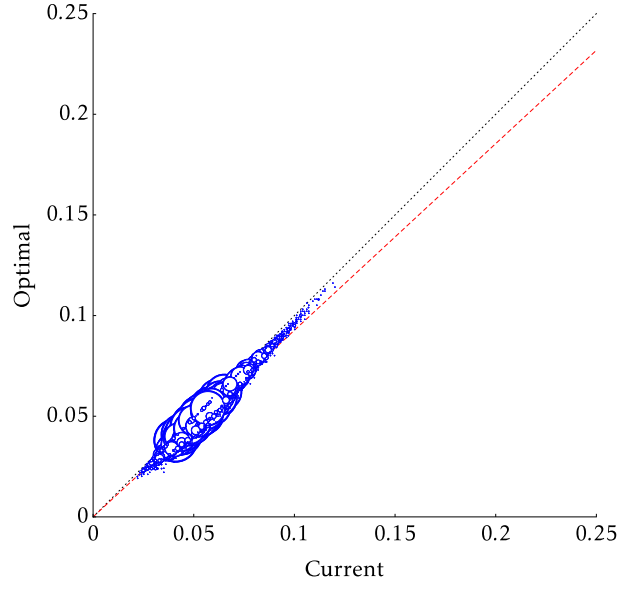


FIGURE 12. Number of constraints: $C = 13$. Red dashed line indicates fitted linear relationship

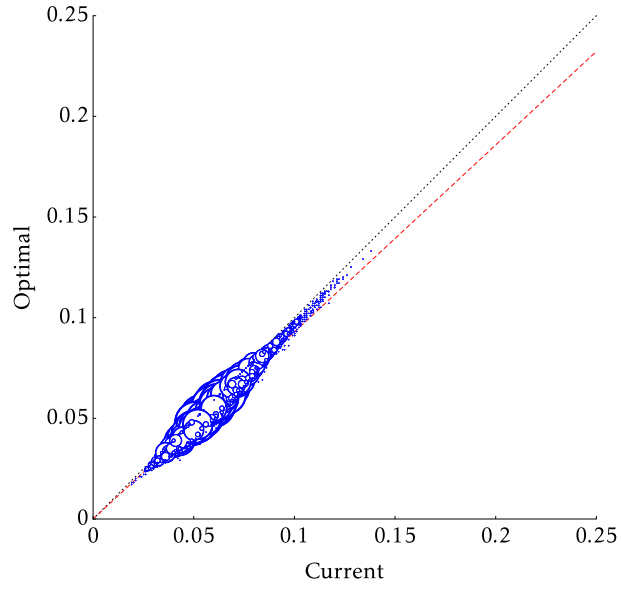


FIGURE 13. Number of constraints: $C = 14$. Red dashed line indicates fitted linear relationship

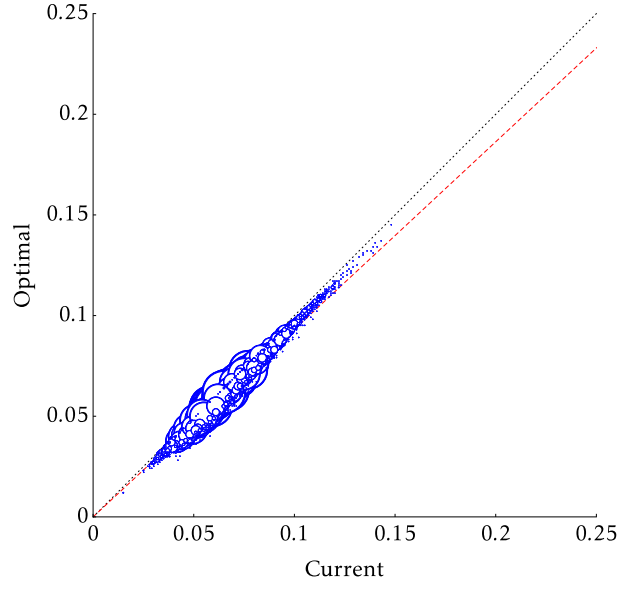


FIGURE 14. Number of constraints: $C = 15$. Red dashed line indicates fitted linear relationship

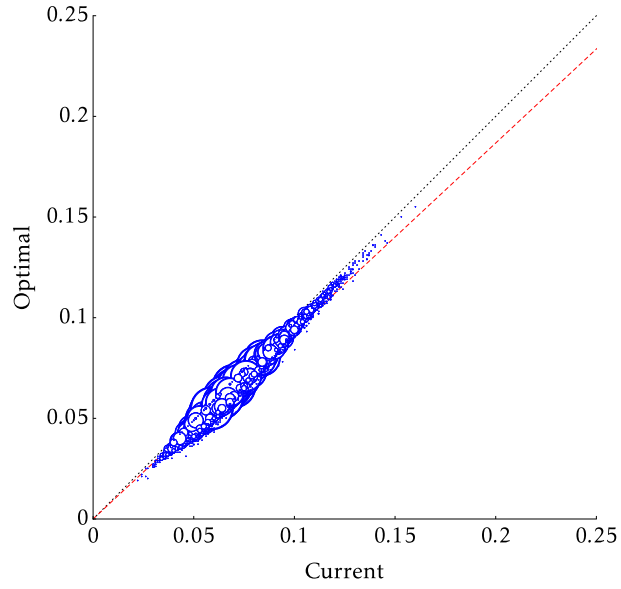


FIGURE 15. Number of constraints: $C = 16$. Red dashed line indicates fitted linear relationship

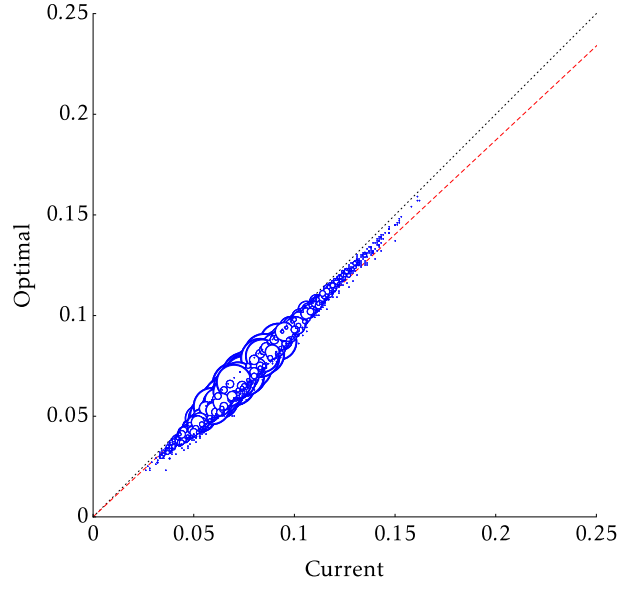


FIGURE 16. Number of constraints: $C = 17$. Red dashed line indicates fitted linear relationship

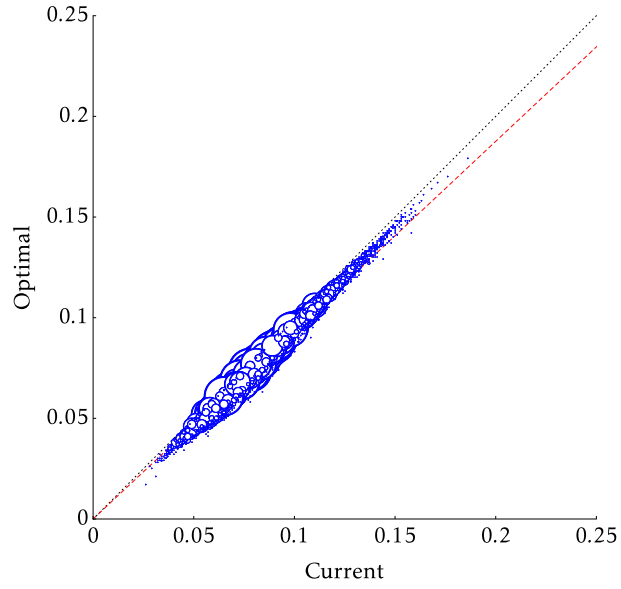


FIGURE 17. Number of constraints: $C = 18$. Red dashed line indicates fitted linear relationship

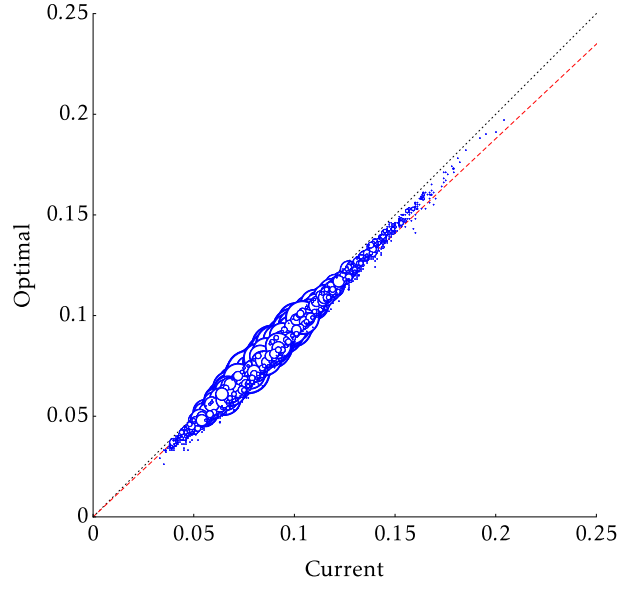


FIGURE 18. Number of constraints: $C = 19$. Red dashed line indicates fitted linear relationship

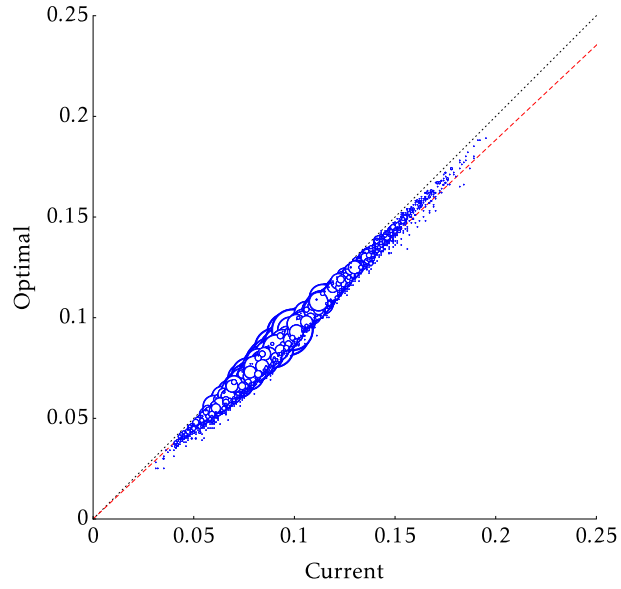


FIGURE 19. Number of constraints: $C = 20$. Red dashed line indicates fitted linear relationship

2.3. Monte-Carlo simulation exercise: Correlated draws.

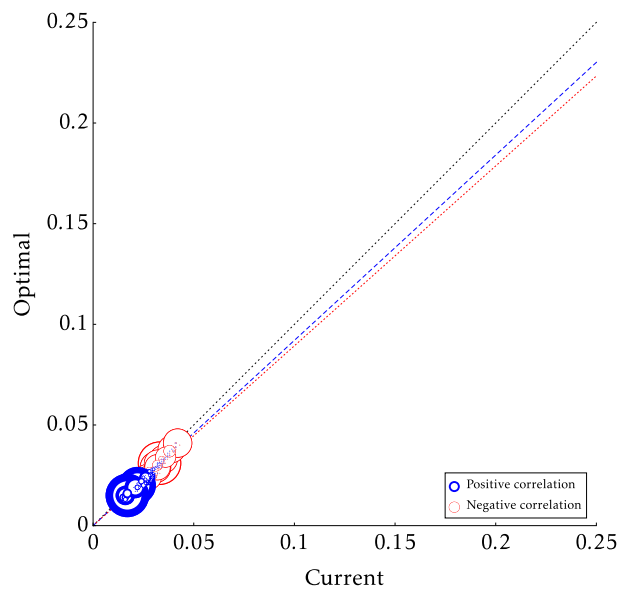


FIGURE 20. Number of constraints: $C = 5$. Red dotted line indicates fitted linear relationship for negatively correlated constraint structures. Blue dashed line indicates fitted linear relationship for positively correlated constraint structures

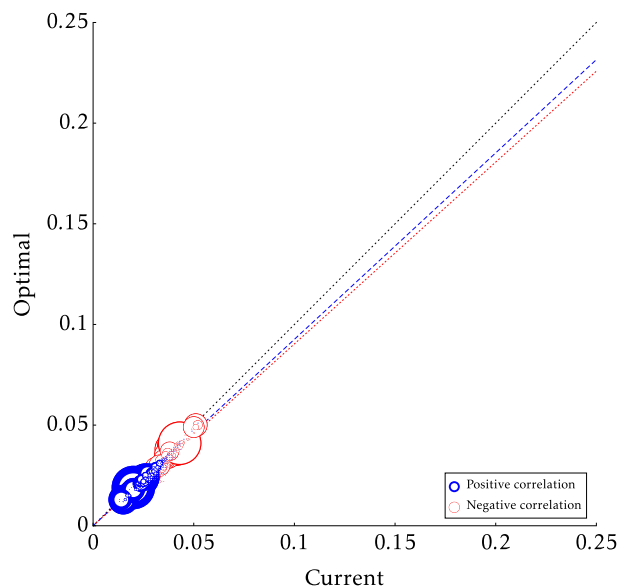


FIGURE 21. Number of constraints: $C = 6$. Red dotted line indicates fitted linear relationship for negatively correlated constraint structures. Blue dashed line indicates fitted linear relationship for positively correlated constraint structures

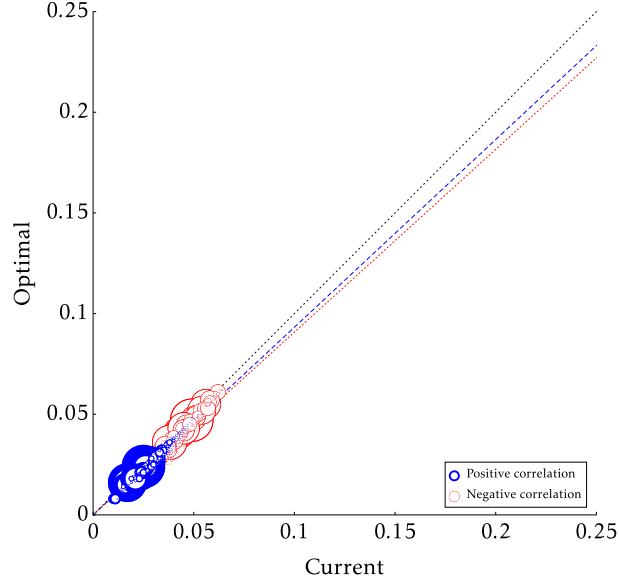


FIGURE 22. Number of constraints: $C = 7$. Red dotted line indicates fitted linear relationship for negatively correlated constraint structures. Blue dashed line indicates fitted linear relationship for positively correlated constraint structures

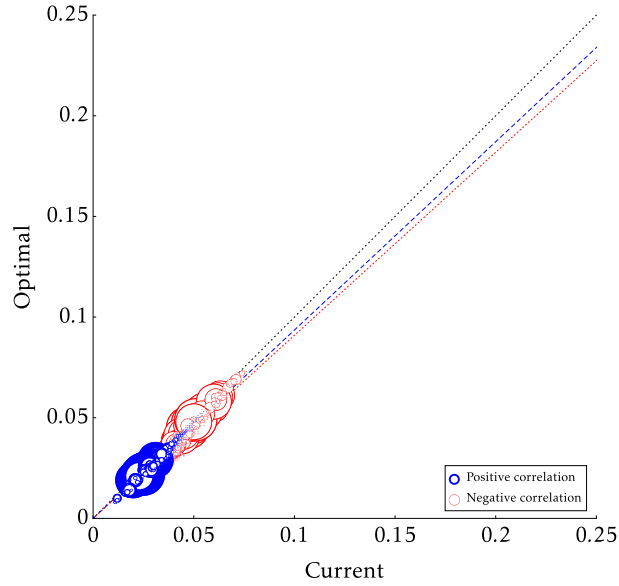


FIGURE 23. Number of constraints: $C = 8$. Red dotted line indicates fitted linear relationship for negatively correlated constraint structures. Blue dashed line indicates fitted linear relationship for positively correlated constraint structures

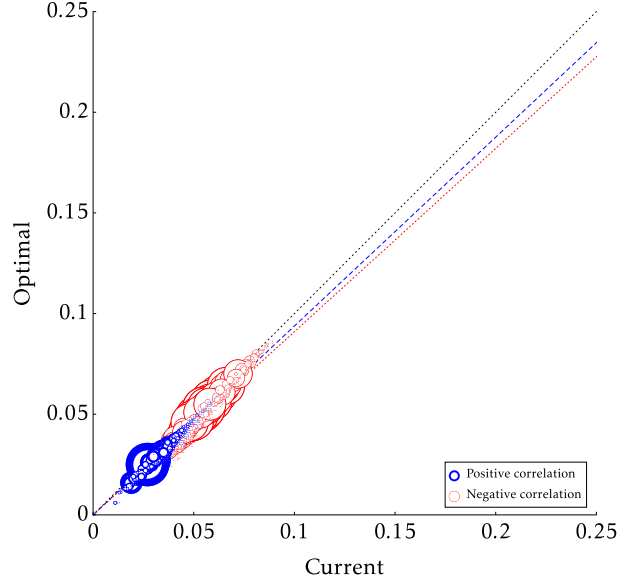


FIGURE 24. Number of constraints: $C = 9$. Red dotted line indicates fitted linear relationship for negatively correlated constraint structures. Blue dashed line indicates fitted linear relationship for positively correlated constraint structures

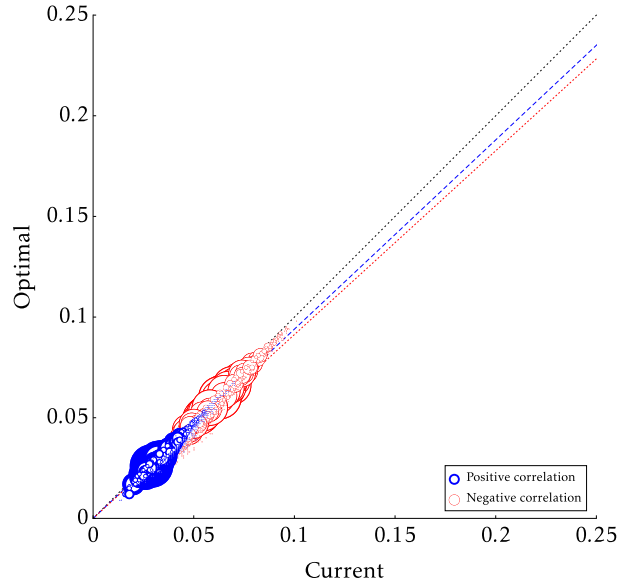


FIGURE 25. Number of constraints:, $C = 10$. Red dotted line indicates fitted linear relationship for negatively correlated constraint structures. Blue dashed line indicates fitted linear relationship for positively correlated constraint structures

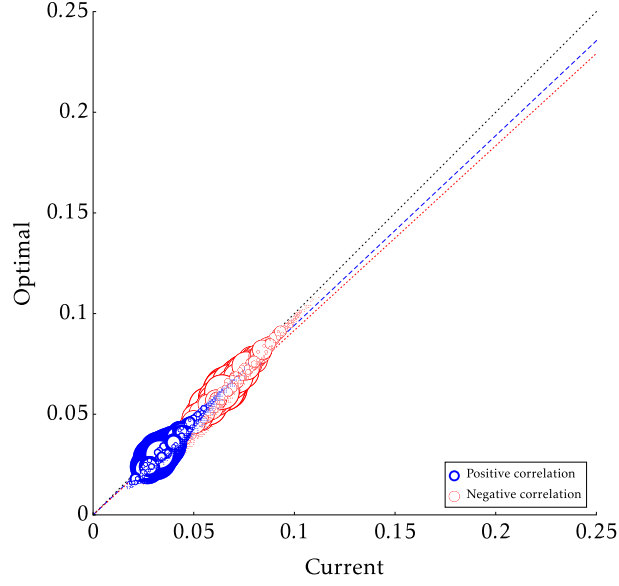


FIGURE 26. Number of constraints: $C = 11$. Red dotted line indicates fitted linear relationship for negatively correlated constraint structures. Blue dashed line indicates fitted linear relationship for positively correlated constraint structures

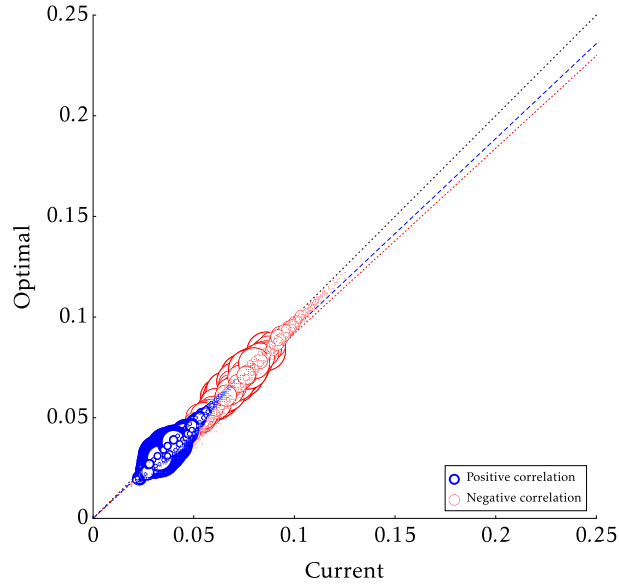


FIGURE 27. Number of constraints: $C = 12$. Red dotted line indicates fitted linear relationship for negatively correlated constraint structures. Blue dashed line indicates fitted linear relationship for positively correlated constraint structures

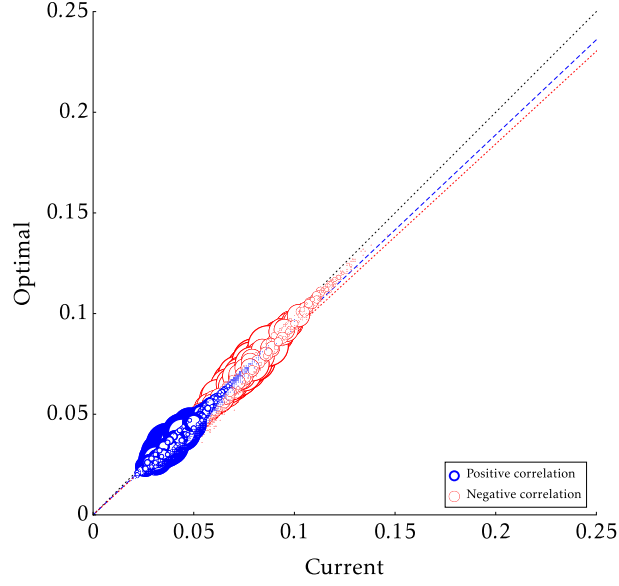


FIGURE 28. Number of constraints: $C = 13$. Red dotted line indicates fitted linear relationship for negatively correlated constraint structures. Blue dashed line indicates fitted linear relationship for positively correlated constraint structures

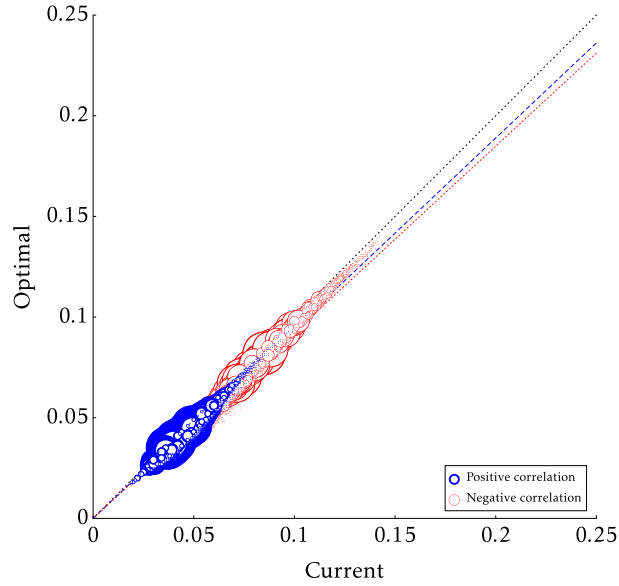


FIGURE 29. Number of constraints: $C = 14$. Red dotted line indicates fitted linear relationship for negatively correlated constraint structures. Blue dashed line indicates fitted linear relationship for positively correlated constraint structures

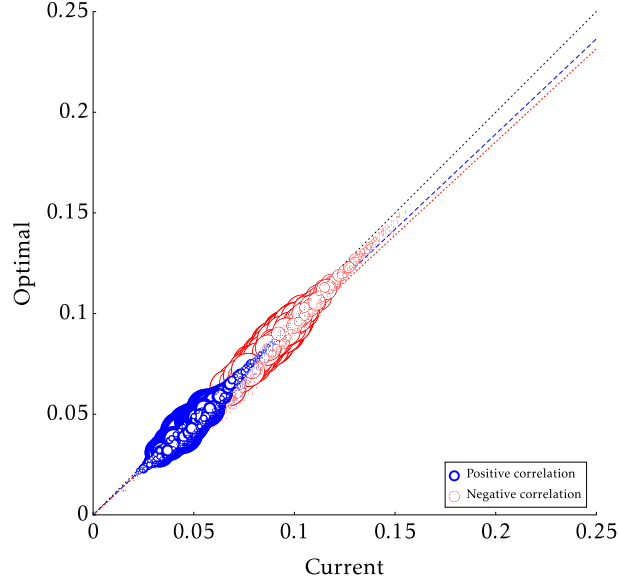


FIGURE 30. FNumber of constraints: $C = 15$. Red dotted line indicates fitted linear relationship for negatively correlated constraint structures. Blue dashed line indicates fitted linear relationship for positively correlated constraint structures

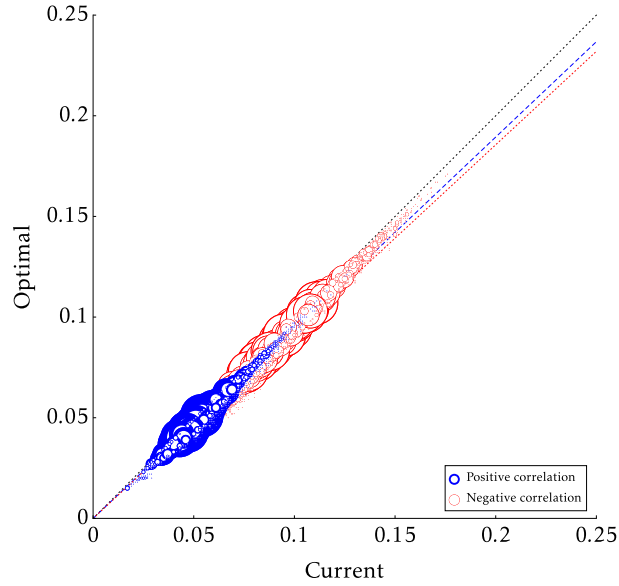


FIGURE 31. Number of constraints: $C = 16$. Red dotted line indicates fitted linear relationship for negatively correlated constraint structures. Blue dashed line indicates fitted linear relationship for positively correlated constraint structures

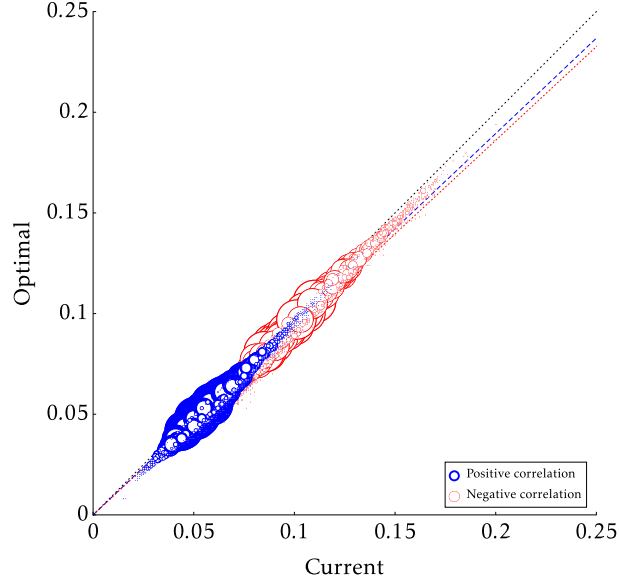


FIGURE 32. Number of constraints: $C = 17$. Red dotted line indicates fitted linear relationship for negatively correlated constraint structures. Blue dashed line indicates fitted linear relationship for positively correlated constraint structures

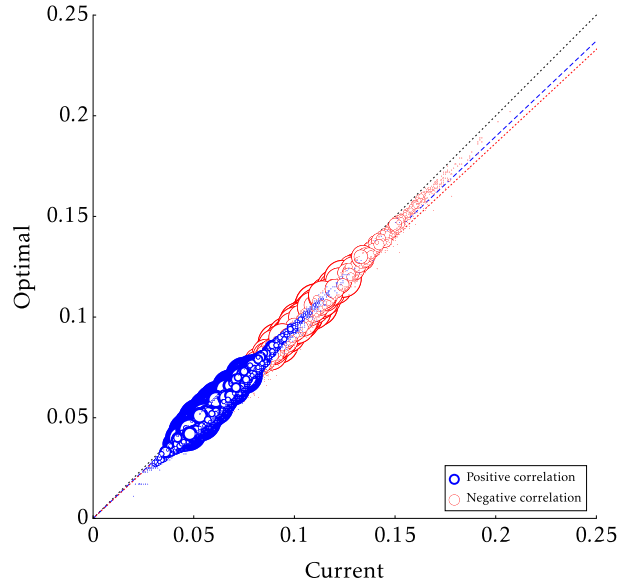


FIGURE 33. Number of constraints: $C = 18$. Red dotted line indicates fitted linear relationship for negatively correlated constraint structures. Blue dashed line indicates fitted linear relationship for positively correlated constraint structures

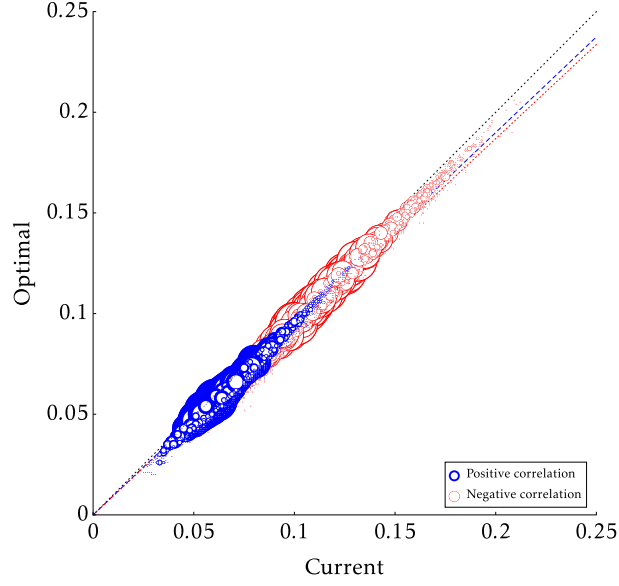


FIGURE 34. Number of constraints: $C = 19$. Red dotted line indicates fitted linear relationship for negatively correlated constraint structures. Blue dashed line indicates fitted linear relationship for positively correlated constraint structures

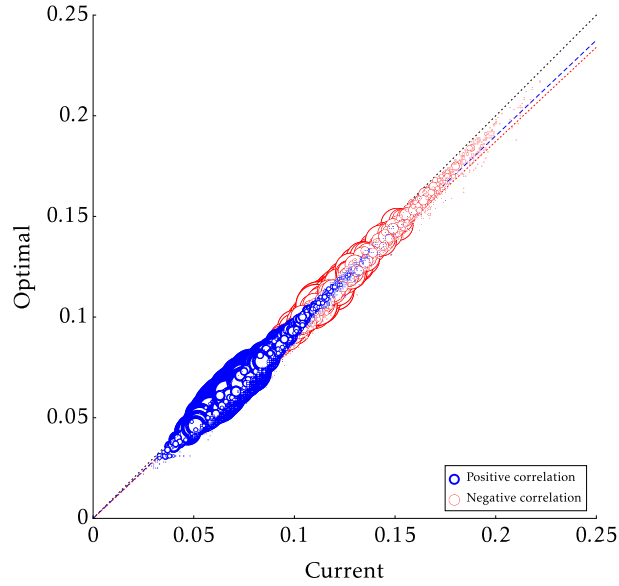


FIGURE 35. Number of constraints: $C = 20$. Red dotted line indicates fitted linear relationship for negatively correlated constraint structures. Blue dashed line indicates fitted linear relationship for positively correlated constraint structures

3. FAIRNESS OBJECTIVE FOR OTHER ASSIGNMENTS: CURRENT VERSUS OPTIMAL

3.1. Six-by-six assignment.

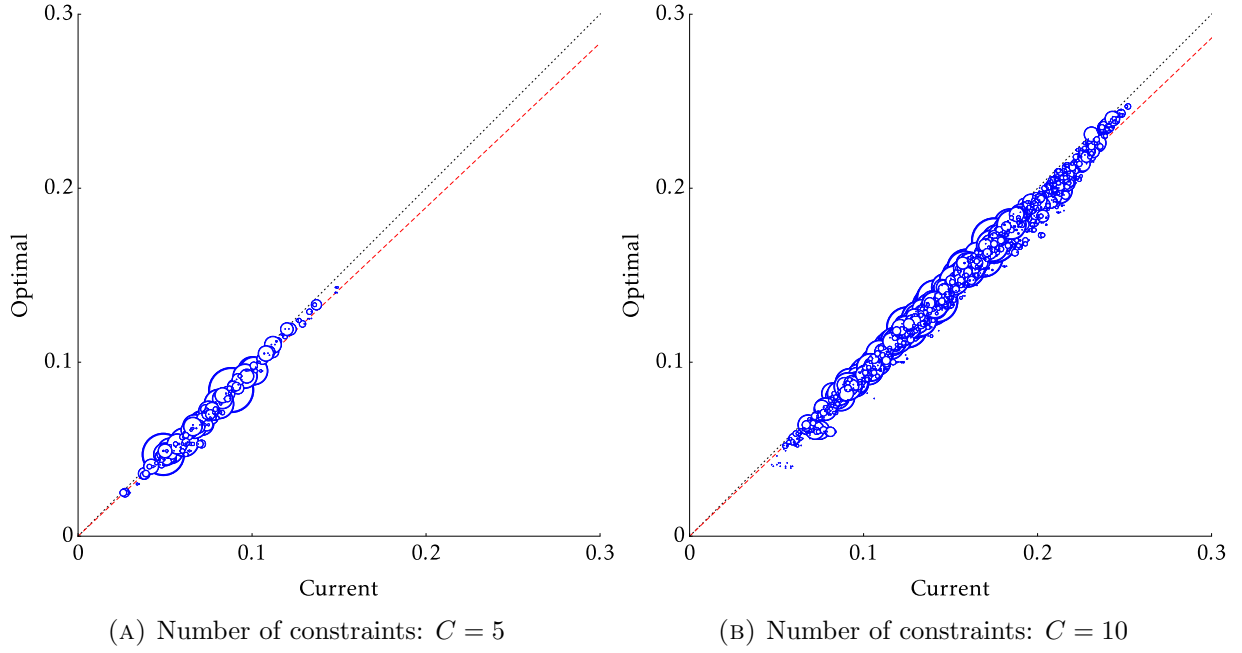


FIGURE 36. Red dashed line indicates fitted linear relationship

3.2. Seven-by-seven assignment.

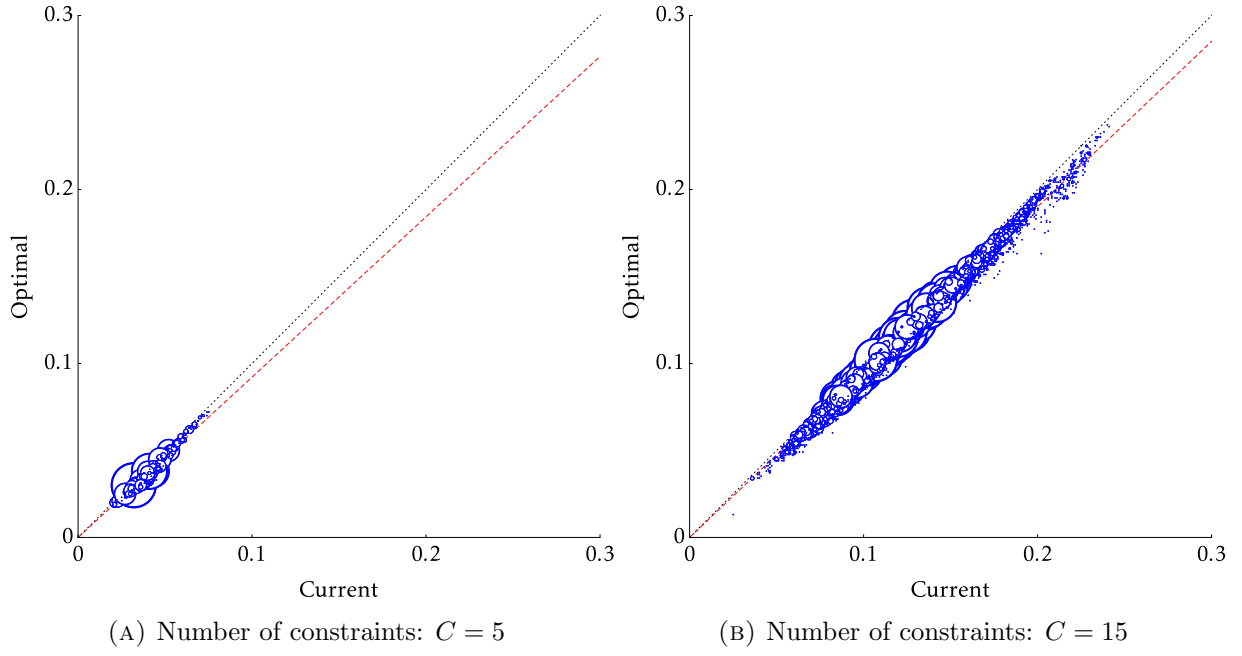


FIGURE 37. Red dashed line indicates fitted linear relationship